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ABSTRACT

This publication is designed to acquaint both mathematics teachers and reading specialists with some of the skills students need to read mathematics effectively and to provide sample activities that may be used as a part of mathematics instruction to help students develop these skills. Topics covered include: Specialized Vocabulary and Symbols, Comprehension, and Study Techniques. Also included is a list of suggested books for voluntary mathematical reading and a list of resource materials for teachers. (Author/MK)

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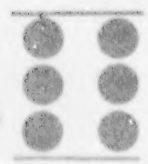
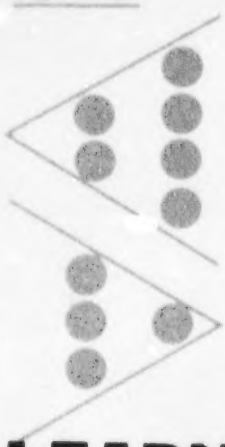


3 = 3



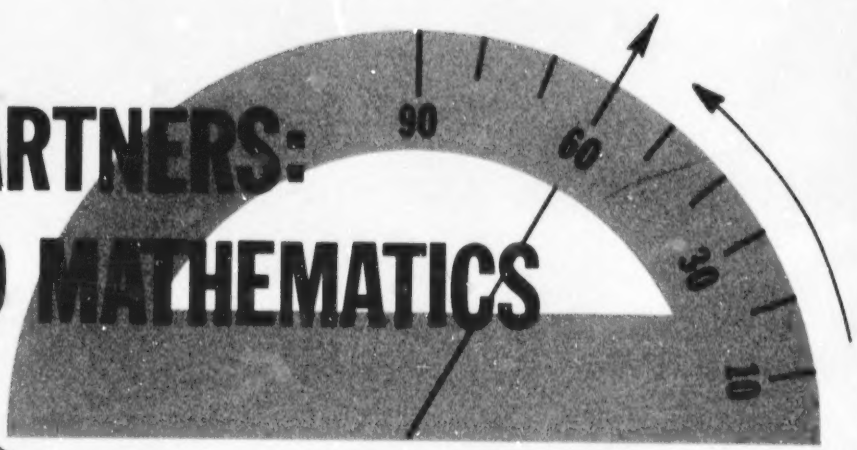
3 > 1

6
- 2½

2 < 4

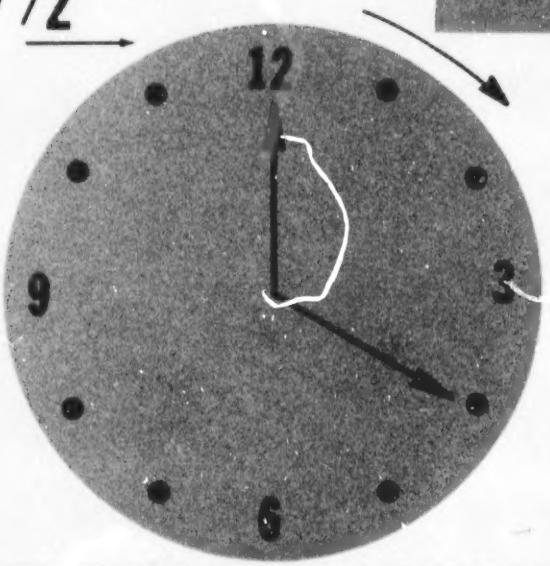
SYMBOL	WORD	PICTURE
3	three	● ● ●
		● ●
½		

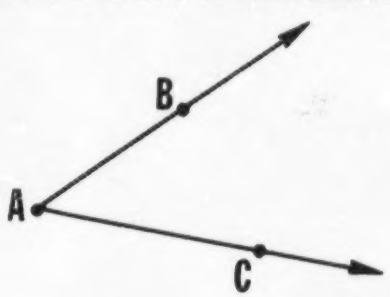
5a + 3a(b + 2b)



LEARNING PARTNERS: READING AND MATHEMATICS

½ + 1





8 ≠ 2
6 ≥ 1
2 > 1
3 < 5

9
- 2

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1979

Carmen worked as a waiter in a department store one summer. One week she worked long hours. On Monday, Tuesday, and Wednesday, she worked eight hours. On the other two days she worked five hours each. At $2.25 per hour, how much money did she earn that week?

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READING AND MATHEMATICS

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FOREWORD

Although reading is fundamental to all disciplines, it is especially important in mathematics, where a mixture of graphic and numerical materials, as well as words, is used. *Learning Partners: Reading and Mathematics* offers a practical approach to the development of reading skills for mathematics taught in elementary and secondary schools. Although the scope of the publication is limited, sample activities are provided to help teachers assist students to develop skills in reading mathematics.

All teachers have a responsibility for seeing that their students develop the skill necessary to read the material required. However, too often the skill of reading is one that is assumed to be known. This guide should encourage teachers to question their previous assumptions about reading and should help them to acquire some knowledge about how to assist their students. It should encourage them to make a new effort at teaching reading skills necessary for improved mathematics achievement.

Alton O. Bowen
Commissioner of Education

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INTRODUCTION

The reading of mathematics involves abstractions, specialized symbolism, and technical terminology. Although many of the word attack skills learned to read narrative material are applicable to the language of mathematics, other specialized skills are also required to read mathematics. Thus, students' effective reading of narrative does not imply they will be equally as effective in reading mathematics.

The mathematics teacher must accept the major responsibility for teaching students to read mathematics, since effective reading in mathematics is closely associated with the development of mathematics skills and concepts. The reading teacher cannot be expected to teach students to read, interpret, translate, and comprehend specialized words and symbols used in mathematics. Meaningful interpretations of these mathematical words and symbols require considerable content knowledge on the part of the student.

As with reading in general, there are no magic techniques that will enable all students to learn the language of mathematics. Mathematics teachers should be aware of some of the factors that make the language of mathematics difficult to read and should provide help to their students in these areas. Among the factors contributing to a student's problems in reading mathematics are the student's interest and motivational level, readiness for reading mathematics, reading level in narrative material, and understanding of the purpose for reading mathematics. The structure of the language of mathematics makes it difficult to read for many students. Differences exist, for example, between the language of mathematics and standard English prose. Such differences include:

- English uses a standard set of 26 symbols. Mathematics uses these symbols plus many nonalphabetic, mathematical symbols.

- The sentence structure in mathematics is different from that of standard English.

- In mathematics, the student does not always read from left to right and down the page.

If teachers would provide necessary information to students in reading the language of mathematics, it would improve student comprehension and increase student success in the development of concepts and computational and problem solving skills. When students develop skills in the reading of mathematics, much of the frustration and anxiety about the subject is eliminated and mathematics becomes less threatening.

This publication is designed to acquaint both mathematics teachers and reading specialists with some of the skills students need to read mathematics effectively and to provide sample activities that may be used as a part of mathematics instruction to help students develop these skills.

The activities provided can be used or adapted for use at different levels of learning.

SPECIALIZED VOCABULARY AND SYMBOLS

The reading of mathematics requires the understanding of a specialized vocabulary and of abstract symbols. Ideas and concepts in mathematics are sometimes difficult, if not impossible, to be adequately communicated in narrative language; therefore, abstract concepts are often explained by using symbols. Specialized words are used that sometimes express broad, complex ideas. These words often cannot be defined simply. They are sometimes new or unfamiliar words or familiar words that take on different or more technical meanings.

Although the language of mathematics is often abstract in nature, understanding of this language can be enhanced through the use of concrete objects, as in experiences with sight, sound, or touch. If students are allowed to work initially with concrete experiences, the mathematical abstractions can become meaningful.

READING MATHEMATICAL WORDS

Reading mathematical words involves:

- developing word meanings by using definitions and examining context clues, prefixes, suffixes, and root words
- interpreting words with special meanings

Teachers sometimes use words orally before students are ready to associate those words with their meanings. To help students associate the spoken word with the written word, teachers can point to words written on the chalkboard or on charts as they are spoken.

Developing Word Meanings

Using Definitions

The meanings of words in the mathematics vocabulary are generally precise, with clearly delineated definitions. It is important, though, that students learning words in mathematics do more than merely memorize and vocalize word meanings. To grasp the materials's meaning, they must develop ideas and concepts represented by the word.

A basic understanding of the concepts involved is required for a student to completely understand a definition. Furthermore, many terms require varying levels of understanding. For example, one could define a square as a quadrilateral with equal

sides and equal angles. However, a student may be able to quote this definition without having a grasp of the meaning. The student may not understand the construction of the square or the relationship of the diagonals or the relationship of sides and area. All of these factors build on the student's complete understanding of the properties of the square.

Many activities can be devised for developing and assessing student's understanding of mathematical words and concepts. The activities that follow are designed for these purposes.

SAMPLE ACTIVITY: Have students make true statements by combining the words on the left with some of the phrases on the right. Students should draw an arrow from the word on the left to the phrase that makes the statement true.

(1) An arc is	(a) a figure having six equal sides
	(b) part of a circle
	(c) the perimeter of a circle

---

(2) A right angle is	(a) an angle whose measure is 90
	(b) an angle whose measure is 60
	(c) an angle whose measure is 180

---

(3) Intersecting lines	(a) cross at exactly one point
	(b) are parallel
	(c) never touch

SAMPLE ACTIVITY: Have students solve a puzzle by filling in the correct words, using the following list of definitions.

Across

1. The result one gets when one adds
2. The number one takes from another number
3. A number to be added to another number or to itself
5. The result one gets when one divides
6. $\{\ldots, -3, -2, -1, 0, +1, +2, +3, \ldots\}$

Down

1. To take away
4. $6\overline{)12}$ The number six in this problem

SAMPLE ACTIVITY: Give the students some words with different meanings and see if any of the nonmathematical definitions relate in some way to the mathematical ones. Have the students express their viewpoints on the relations of meaning and explain their particular opinions.

SAMPLE ACTIVITY: Have students create their own mathematics dictionary. Words and definitions can be put on cards and later transferred to a booklet. Keeping words and definitions on cards at first will allow the students to be able to put them in alphabetical order after the collection is completed. Suggest that students draw their own illustrations.

Using Context Clues

Clues from pictures, diagrams, examples, and familiar words in the sentence can aid students during the process of reading. It is important that students develop an ability to use context clues if they are to read mathematics with meaning. The following activities are designed to help students learn to search for context clues.

SAMPLE ACTIVITY: Construct exercises such as the following, leaving blanks for selected words or symbols. The students should fill in the blanks with correct words or symbols (synonyms should be allowed).

The distance around a circle is called the _____ of the circle. The diameter of a circle is the length of a segment with _____ on the circle and that passes through the _____ of the circle. The circumference divided by the _____ is the same for all circles. This quotient is symbolized by the Greek letter _____.

SAMPLE ACTIVITY: Have students explain the meaning of the italicized words. Encourage them to use the context to determine their meaning.

(1) $15 \div 3 = 5$. We call the numeral 5 a *quotient*.

(2) $20 - 5 = 15$. We call the numeral 15 a *difference*.

(3) Fractions like 1/2, 2/3, 3/5, and 4/5 are called *proper* fractions.

(4) Fractions like 3/2, 4/3, 7/3, and 5/4 are called *improper* fractions.

Using Prefixes, Suffixes, and Root Words

The meaning of certain mathematical words can often be determined by observing the meanings of prefixes, suffixes, and root words. For example, the words *add, adding, addend, additive, adds,* and *addition* share a common root. Students will know many of these prefixes, suffixes, and root words, and they can be guided into adapting them for mathematics.

Teachers can direct students to become sensitive to the roots, prefixes, and suffixes as new words are introduced. The activities that follow can improve student awareness of word meanings.

SAMPLE ACTIVITY: Ask students to name as many words as they can that begin with the following:

mid	poly	circum
de	sub	
ex	sum	
bi	tri	

Then have them identify the prefixes that are mathematical.

SAMPLE ACTIVITY: Have students match each word on the left with a descriptive word on the right which could help define the word.

(1) quadrilateral	_____	(a) five
(2) exterior	_____	(b) around
(3) semicircle	_____	(c) eight
(4) pentagon	_____	(d) two
(5) circumference	_____	(e) four
(6) octagon	_____	(f) equal
(7) triangle	_____	(g) half
(8) bisect	_____	(h) three
(9) equilateral	_____	(i) outside

SAMPLE ACTIVITY: Use flash cards on which have been printed mathematical roots and prefixes. Have the students name as many mathematical words as they can that contain the roots and prefixes. Some examples could include the following: *add, multi, sub, num, equa, equi, circum, divi.*

Interpreting Words with Special Meanings

Many words used in a mathematical context take on a different or specialized meaning. A few examples of words with specialized meanings include *solution, plane, volume, power, root,* and *group.* As previously mentioned, words in mathematics generally have precise definitions. However, there are words, such as *degree, square,* and *base* that have multiple meanings in mathematics.

The activities that follow involve words which have specialized meanings in a mathematical context.

SAMPLE ACTIVITY: Write a sentence using each of these words in a mathematical context:

(1) angle	(6) square
(2) degree	(7) set
(3) plane	(8) volume
(4) odd	(9) solution
(5) base	(10) power

SAMPLE ACTIVITY: Help the student read a chapter in a mathematics book and look for specific words and their meanings. Then have the students match each word with a meaning to show how it was used in the chapter.

WORDS	POSSIBLE MEANINGS
(1) square	(a) super conservative person
	(b) the product of a number multiplied by itself
	(c) an unopened cotton flower
	(d) a rectangle with sides of equal length

(2) degree

(a) the largest exponent of a variable in a polynomial

(b) unit of measure for temperature

(c) unit of an angular measurement

(d) a step in a process

---

(3) range

(a) a cooking stove

(b) grassland used for livestock grazing

(c) distance a bullet travels

(d) the set of second elements in a set of ordered pairs

(e) difference between the lowest and highest scores in a group of scores

---

(4) base

(a) foundation

(b) main ingredient

(c) military installation

(d) the factor on which an exponent is placed

READING MATHEMATICAL SYMBOLS

A symbol in mathematics may be a character, abbreviation, or mark which represents an idea, concept, expression, quantity, or operation. Many mathematical symbols, such as the "less than" sign ($<$) or the "equal" sign ($=$), are entirely different from customary reading symbols. Others, however, such as the "x" to indicate multiplication or the " – " to indicate subtraction, are standard reading symbols that have specific meanings in mathematics.

The following are examples of symbols frequently used: +, =, { }, –, Δ, 1, 2, 3, ... In order to work successfully with symbols, students need to be able to:

- associate a word or phrase with a symbol
- express an idea in objects, pictures, words, and symbols
- be aware that there may be more than one meaning for some of the symbols

Associating a Word or Phrase with a Symbol

Example: symbol - " $>$ " phrase - "greater than"

SAMPLE ACTIVITY: Develop a set of cards with phrases of mathematical symbols. Have a student come to the front of the class and give him or her one of the cards. Do not allow the rest of the class to see what is on the card. The student then reads aloud what is on the card. The other students write the symbols for what they hear. Students can compare their answers.

$3 + 2$	$\frac{1}{7}$	$\frac{2}{3} + 4$
$\frac{2 + 1}{7} + (1 \times 4)$	$\frac{(X + Y)^2}{(X + Y)}$	$\text{LOG}_3 X = 2$

Note: Parentheses need not be silent. For greater clarification, have the student say, "Three minus, parenthesis open, two plus seven, parenthesis closed", for 3 - (2 + 7).

SAMPLE ACTIVITY: Have students place their chairs in a row to make a train and select a conductor. The conductor stands next to the student in the first seat. Flash a card with a mathematical symbol on it. The conductor and the passenger try to identify it. If the conductor identifies it first, he or she remains the conductor and moves to the side of the next passenger. If the passenger gets the word first, he or she becomes the conductor. The challenge to the conductor is to see if he or she can go through the entire train. The passengers are trying to stop the conductor by identifying the symbols.

SAMPLE ACTIVITY: Divide the class into two teams and give one team a set of flash cards both with mathematical words that can be abbreviated and with mathematical abbreviations. Four corners of the classroom may be used as bases. One student goes to homeplate. The pitcher holds up a flash card. If the card has an abbreviation on it, the batter says the word; if the card has a word on it, the batter writes the abbreviation on the board. The pitcher isn't allowed to alter the order in which the cards are stacked. If correct, the batter goes to first base and the next batter comes up. Each batter advances as each teammate gets a hit. A run is scored when a student crosses homeplate. When a team misses three, that team is retired and the other team takes over. The team that has the highest score wins the game.

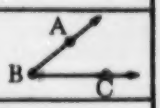
Expressing an Idea in Objects, Pictures, Words, and Symbols

Example: idea - "the number two" words - "two objects"

SAMPLE ACTIVITY: Pictures can be drawn to interpret mathematical symbols. Along with the pictures, words can be used for symbols. Demonstrate this idea by having students do the following exercise.

Complete:

Example:

Symbol	Word	Picture
3	three	•••
		••••
$\overleftrightarrow{AB}$		
		
½		

Note: To use this activity, the teacher could fill in only one of the columns. Or, as shown above, only one item from each column could be filled in with the students supplying the missing information.

SAMPLE ACTIVITY: To teach the meanings of symbols and relationships, make a set of cards containing statements such as the following: $3 < 5$, $2 > 1$, $5 \geq 3$, $8 \neq 2$. The students take the cards and lay some small objects such as dry beans, corn kernels, or buttons on each side of the symbols to match the numbers.

Example: $3 < 5$

••• •••••

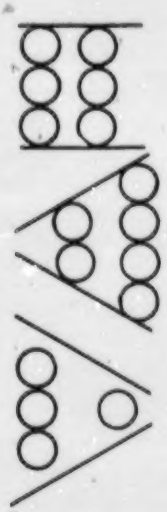
SAMPLE ACTIVITY: Grouping symbols are used for clarification in mathematical expressions. Have students tell the order in which they would perform the operations in each exercise.

Example: (1) $5 + 3 \cdot 4$ multiply, add
(2) $36 \div 3(4)$
(3) $5a + 3a(b + 2b)$

Also, have students match grouping symbols:

$$\underline{(\{[(5 + 4) - 2) + (3 + 1)]\} + 2)}$$

SAMPLE ACTIVITY: Have students work with a pictorial development of the meanings of $=$, $<$, and $>$, as well as the words.



$3 = 3$ "is equal to"

$2 < 4$ "is less than"

$3 > 1$ "is greater than"

Become Aware of Multiple Meanings for Some Symbols

SAMPLE ACTIVITY: Ask the students to discuss the meaning of " – " in each of the following

(1) $\overline{AB}$

(2) $8 - 2$

(3) $^{-}4$

(4) $3.\overline{3}$

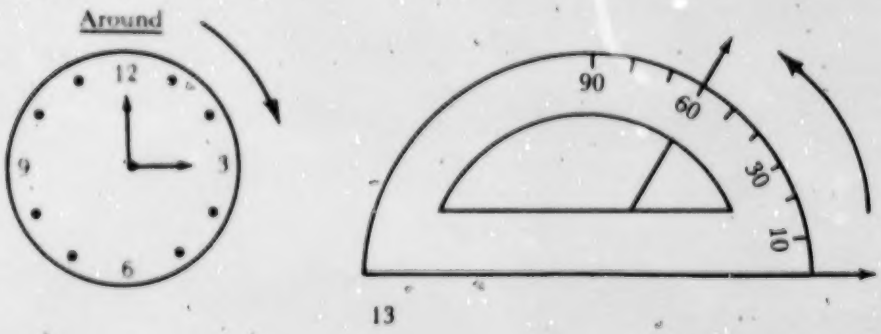
DIRECTIONALITY OF MATHEMATICAL SYMBOLS

Directionality and spatial relationships often cause difficulty in the reading of mathematics. For example, elementary teachers may have students who find confusion between 65 and 56. Algebra teachers may have students who confuse $5x$ and x^5. In each case, the meaning of the 5 has changed according to its position. Furthermore, there is not necessarily a one-to-one relationship between the mathematical symbols and the spoken word. For example, x^5 can be read as the fifth power of x.

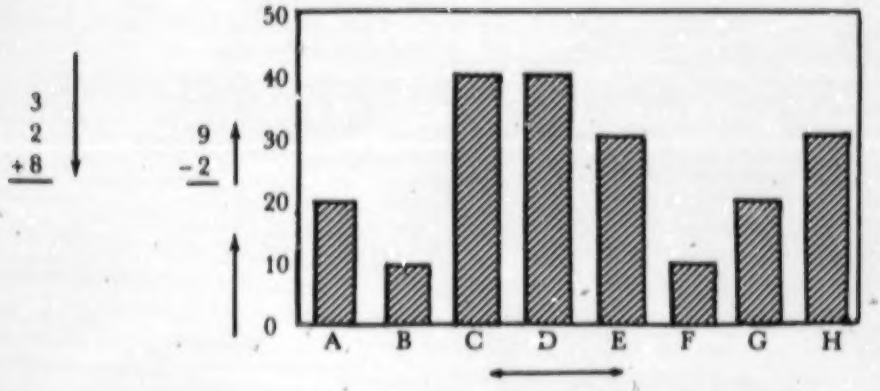
Because of the nature of our place value system, one skims a number in a back-and-forth motion to read it. To read the number 748, for example, one must note that there are three digits, indicating a number in the hundreds. One then reads the first digit and follows it with a hundred. One then observes the 4 in the tens place, indicating 40. Then the 8 is observed in the ones column to be added to the 40. Reading 748 as seven hundred forty-eight, therefore, is a more complex activity for students than we often realize. Pronouncing a phrase such as $(x + 5)^3$ as the cube of the sum of x and 5 requires a similar back-and-forth motion which is sometimes difficult for older students.

The following examples illustrate some of the directions and eye movements involved in reading mathematics material.

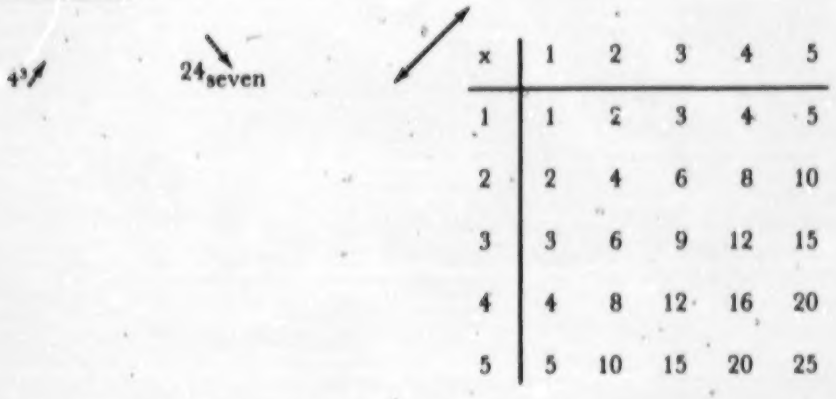
Examples:



Top to Bottom - Bottom to Top



Diagonally



x	1	2	3	4	5
1	1	2	3	4	5
2	2	4	6	8	10
3	3	6	9	12	15
4	4	8	12	16	20
5	5	10	15	20	25

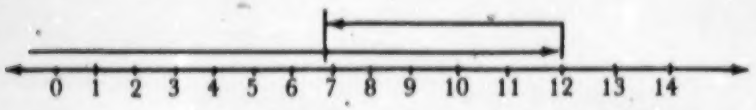
Forward - Backward

The item $3\overline{)17}$ may read three into seventeen (left to right), or seventeen divided by three (right to left).

Put in the commas (←), then read aloud the number (↔).

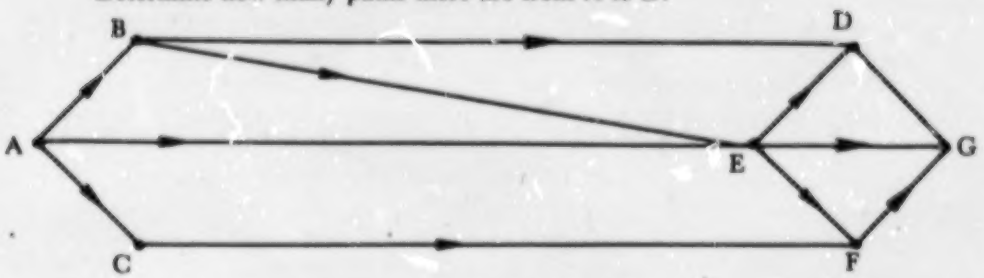
1000000

Use the number line to find 12-5



Multiple Directions

Determine how many paths there are from A to D.



The following activity is designed to make students more aware of the directionality of reading mathematics.

SAMPLE ACTIVITY: Make a wall chart that contains some symbols that may create a problem in reading. Guide the students through a discussion of the directions to be taken in reading the symbols. With a colored pen, draw arrows to indicate the directions to be taken. In addition to drawing arrows, write the expression in words. Discuss the chart, and leave it up for further reference.

Examples:

(1) 5^2 "the square of five"

(2) 5^2 "five squared"

(3) $3x$ "three times x"

(4) $5 : 2$ "five is to two"

(5) $4\overline{)12}$ "twelve divided by four"

(6) $\frac{1}{2} + 1$ "one-half plus one"

(7) $6 - 2\frac{1}{3} = \square$ "Six minus two and one-third is equal to what number?"

(8) $7 - \square = 3$ "What number subtracted from seven leaves three?"

COMPREHENSION

Comprehension is the act of getting meaning from a printed or spoken language. Two types of comprehension, literal and interpretive, are important in the understanding of mathematics. Literal comprehension is the understanding of the direct meaning of a word, phrase, sentence, or longer selection. For example, noting important details is a literal comprehension skill. Interpretive comprehension is the understanding of the implied meaning. Locating the main idea or major details when they are not directly stated would be an example of interpretive comprehension. In a mathematics problem, an operation is sometimes implied rather than directly stated. Students must be able to comprehend implied information so they can recall or locate information to help them understand a new algorithm or concept.

In order to comprehend mathematics, students need to be able to:

- understand main ideas and important details
- follow directions
- interpret graphic materials
- read mathematical sentences

UNDERSTANDING MAIN IDEAS AND IMPORTANT DETAILS

Students need to be able to identify and to understand main ideas and supporting details and to determine the relationships between the two. The main idea may appear at the beginning, in the middle, or at the end of a page. It may be found by looking at diagrams, tables, examples, pictures, etc. In some situations it may be in italics, in boldface type, or underlined. The following activities help students become aware of the main idea.

SAMPLE ACTIVITY: Direct students to turn to a page in the mathematics book and answer specific questions. Or write a passage and questions, such as the following, on the chalkboard.

(1) What is the main idea?

(2) Name several supporting details.

(3) How are the main idea and details related?

SAMPLE ACTIVITY: Ask students to answer these questions as they read the passage below for meaning.

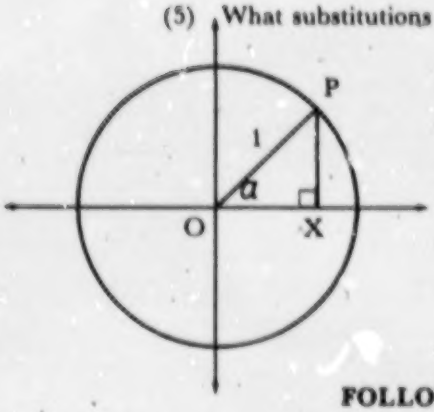
(1) Put a check where you first knew how to find the trigonometric identity.

(2) How did you find this identity?

(3) What previously learned ideas did you have to recall?

(4) Where did you discover you were going to use trigonometry?

(5) What substitutions in the formula did you have to do?



This is a diagram of a circle of radius 1. Look at the right triangle, $\triangle OXP$. State the formula for finding the length of the hypotenuse $\overline{OP}$ _____.

Now use $d\overline{PX} = \sin a$ and $d\overline{OX} = \cos a$ to find the trigonometric identity: $1 =$ _____.

FOLLOWING DIRECTIONS

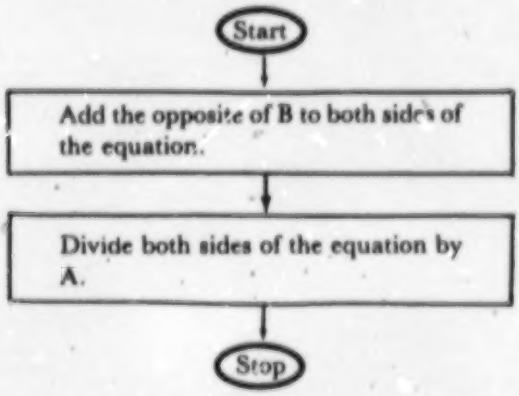
Following directions is a skill that requires students to read and to perform the activities they are instructed to do.

Students need certain abilities to follow directions. For example, the word *simplify* is sometimes used as a direction. This word could mean to combine like terms or to reduce a fraction to lowest terms. Teachers should emphasize the importance of understanding directions, so that students do not purposefully avoid them. Teachers should be able to clarify directions and walk students through them. They should avoid giving students directions more complicated than the students' ability to follow them. A general rule of thumb for giving directions is that third graders grasp one-step directions, fifth graders grasp two-step directions, and eighth graders grasp three-step directions.

The activities below emphasize following directions.

SAMPLE ACTIVITY: In addition to guiding students through a direction, help the students learn to follow directions by having them rewrite step-by-step directions in their own words.

SAMPLE ACTIVITY: Flow charts are often useful when activities need to be performed in a particular sequence. (1) Have students design flow charts to describe a procedure. For example, have them construct flow charts that describe how to divide whole numbers. (2) Provide flow charts to help illustrate directions. For example, to solve equations of the form $Ax + B = C$:



SAMPLE ACTIVITY: Often students need to be reminded to read and follow directions carefully. To keep them aware of this, periodically give the students a list of sample directions to follow. The following directions may be used:

(1) Open your mathematics book to page five.

(2) Write, on your paper, the first word that is printed at the top of page five.

(3) Read the first problem on page five.

(4) Write the answer to the first problem.

(5) Close your book.

(6) Draw two triangles on your paper.

(7) Color one triangle blue.

(8) Pass in your papers.

Note: An activity such as this should be increased in complexity as the students become more sophisticated in following directions.

INTERPRETING GRAPHIC MATERIALS

Mathematical ideas are often compacted through the use of graphic materials, such as tables, graphs, or charts, and the reading and interpreting of these materials play an important part in the reading of mathematics. Students should begin developing skills in reading graphs at a simple level and progress gradually to more complex situations.

When reading graphic materials, the student should first survey to determine what the graphic is about, how it is organized, and what information is supplied. Following this overview, the graphic can be studied in more detail for the specific information required.

The activities that follow are designed to provide practice to students in reading graphic materials.

SAMPLE ACTIVITY: Have students make their own graphs. Small groups can graph different characteristics of the class. The following characteristics may be used:

- student heights
- student weights
- color of eyes
- types of books liked
- types of movies liked
- favorite school subjects

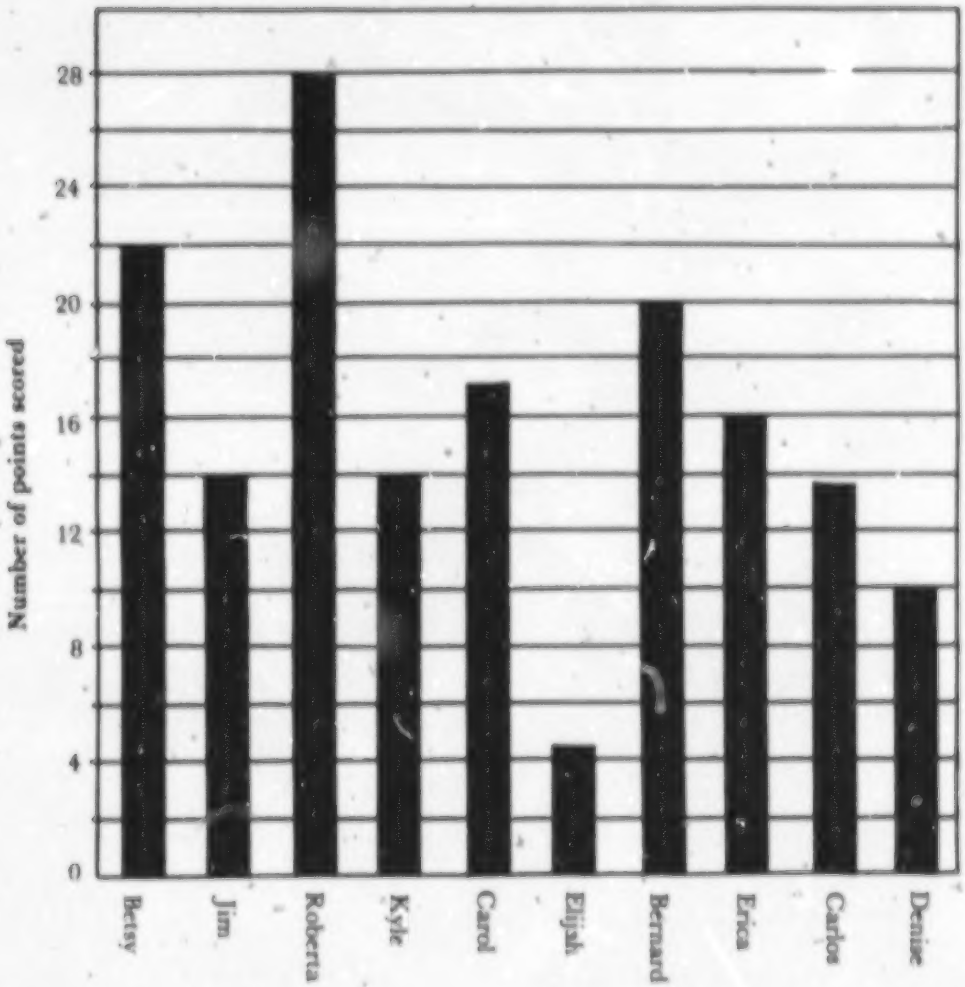
SAMPLE ACTIVITY: Give the students a graph, such as the one below, and ask specific questions about it. For example:

Underline the statements which are true:

- Denise made more points than Elijah.
- Kyle made fewer points than Carlos.
- Three of the players made 14 points each.
- Betsy made fewer points than Bernard.
- Roberta made more points than anyone.

- There are 12 players shown on the graph.
- Carol made 17 points.
- It is possible to show 7½ points on the graph.

BASKETBALL SCORES



SAMPLE ACTIVITY: Give the students a graph, such as the one below, and ask specific questions about it. For example:

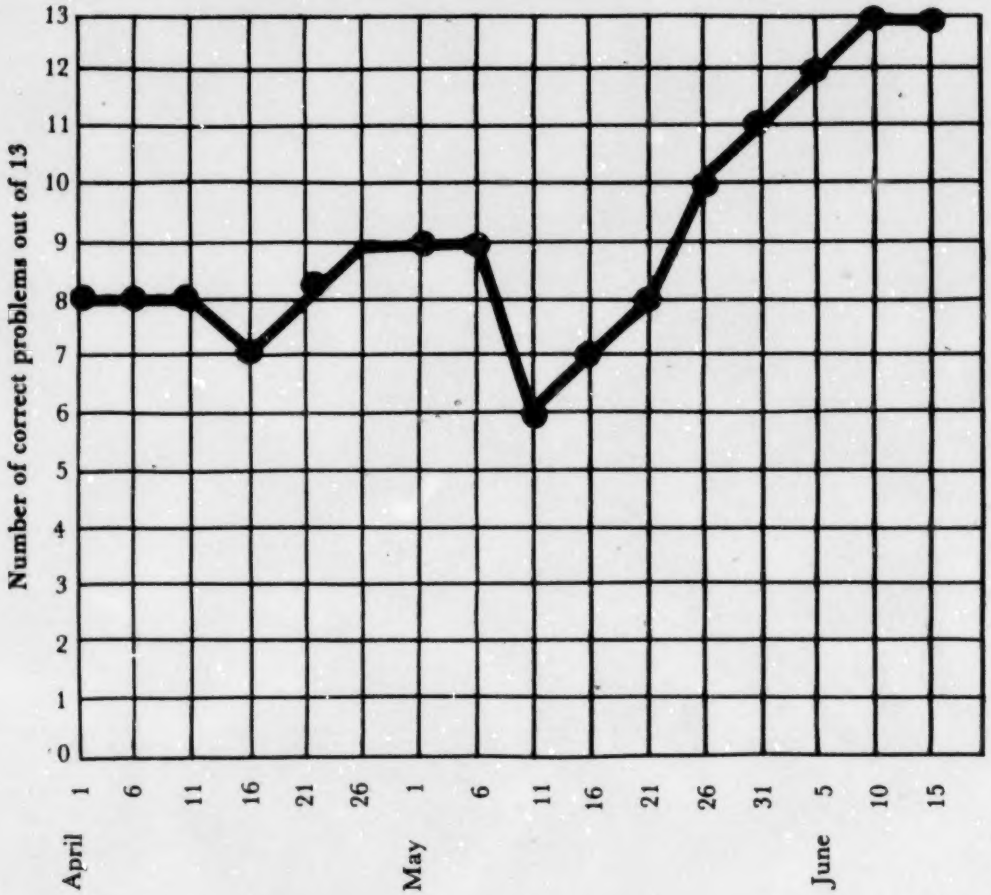
(1) What is the purpose of this graph?

(2) On what date did Carol get the least number of problems correct?

(3) On what date did Carol get 12 problems correct?

(4) On what date did Carol's score double a previous score?

CAROL'S MATH SCORES



READING MATHEMATICAL SENTENCES

At an early stage of their mathematical development, students must learn to read and understand mathematical sentences. The reading of these sentences involves recognizing and pronouncing words and symbols and attaching meaning to the total sentence. A mathematical sentence can be modeled in the following way: symbols that name things, followed by a symbol that names a relationship, followed by symbols that name things. In the sentence $9x + 3 \leq 4x + 2$, for example, $9x + 3$ and $4x + 2$ are symbols naming things, and $\leq$ is a symbol that names a relationship. Students must be able to interpret all the symbols involved and then put all the symbols together to attach meaning to the entire sentence.

The following activity shows some types of questions that can be asked to assess student understanding of mathematical sentences.

SAMPLE ACTIVITY: Have the students answer specific questions concerning mathematical sentences. For example:

$6 + 7 = 13$

(1) What is the operation?

(2) What does the 1 tell you in the above sentence? The 3?

(3) How much more than 6 is 13?

(4) Write the sentence using words. Is it a true sentence?

(5) Write the sentence in words, reading from right to left. Does it still sound true?

Note: When we write $1 + 3 = \square$ and do not explain it, we sometimes give the idea that this is the only order in which this sentence can be written. Many times when students see $4 = 1 + 3$ they are confused about the order in which it should be read. This causes them to think that this is an incorrect form. The activity which follows can be used to help young students understand the flexibility in the sentence order.

SAMPLE ACTIVITY: Make cards (8'' x 11'') with strings attached so students can wear them. On each card write one of the following symbols (if appropriate for your class): 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, +, -, =, <, >. Make several cards with each symbol. The students can be called on, one at a time, to choose a card to wear. They can stand facing the class to build a living number sentence. If students do not choose cards that match, they have to wait for a chance to fit in the sentence.

READING AND SOLVING WORD PROBLEMS

Many students experience difficulty when they are asked to solve word problems, although they are often able to solve the problems without difficulty when someone performs the translation for them. In order to experience success with solving word problems, students need to:

- be prepared for reading word problems
- be able to analyze the information in word problems
- be able to translate and use signal words
- be able to identify sufficient, insufficient, and extraneous information in a word problem
- be able to employ some strategy, someone else's or their own, that will help them solve word problems

Preparing for Reading Word Problems

In reading, the word readiness is used to refer to a student's preparedness to understand beginning reading instruction or a reading activity at any level of instruction. Readiness for reading must be present in all reading activities from kindergarten through adult education. Likewise, in mathematics, readiness to perform a given task is needed. If students cannot adequately read and understand a word problem, they must be given help to do so. A mathematics teacher cannot afford to ignore the reading levels of the students nor of the material to be read. Every attempt should be made to facilitate the learning process.

Another consideration of readiness is to be sure the students can perform the necessary arithmetic and algebraic operations required to solve the problem. Even if they have analyzed the problem, they cannot solve it unless they can perform the required operations.

Word problems should be assigned in small quantities, and effective teachers will keep this in mind when making assignments. Teachers should also attempt to arrange problems according to difficulty. Early success tends to motivate students' progress.

The following activity provides one way of helping students develop word problem readiness.

SAMPLE ACTIVITY: Have students form small groups, make up problems of their own, and state them in their words.

Analysing Word Problems

Students need to be able to state meanings, relationships, and needed operations in order to solve a word problem. They must be able to state the problem situation correctly in their own words and to determine what is being asked. They should be able to follow ideas in a sequential manner.

In an effort to build skills in analyzing word problems, the teacher can use an activity, such as the following, in which the students are required to respond to a list of questions about a problem situation. If students find it hard to organize their thoughts, it is the responsibility of the teacher to determine if the material is too hard or if students need help with organization.

SAMPLE ACTIVITY: Have students read some verbal problems and supply the information asked for in the following chart:

	What Information is needed?	What terms need defining?	What operation is needed?	What is the order of operations?	What variables are given?
Problem 1					
Problem 2					
Problem 3					
Problem 4					

The four activities that follow are also designed to build skills in analyzing word problems.

SAMPLE ACTIVITY: Give students word problems and encourage them to ask as many questions as they can about the problems. Have students use the previous activity as a model in making their own activity.

SAMPLE ACTIVITY: Have students read verbal problems and list the operations they must use to solve the problems.

SAMPLE ACTIVITY: Select a word problem that requires the students to think in sequence. Each sentence should be written on a separate cardboard strip and put in an envelope. The student can then open the envelope, take out the strips, and arrange them in order. If the teacher writes the numbers on the back of the strips, the students can check their own work. These strips can be made to use with a flannel board, or they can be pinned on a cork board. If all students in a class are having problems in this area, the teacher can give each one an envelope; when they finish, the envelopes can be exchanged.

Suggested problem: Carmen worked as a cashier in a department store one summer. One week, she worked long hours. On Monday, Tuesday, and Wednesday, she worked nine hours per day. On Thursday, she worked eight hours. On the other two days she worked five hours each. At $2.25 per hour, how much money did she earn that week?

SAMPLE ACTIVITY: Let students write their own word problems that will use ideas of different operations. For example, word problems can be written that deal with addition, multiplication, division, and subtraction.

Students often experience difficulty in translating written word problems into mathematical sentences. Key words or phrases can be translated into mathematical symbols to aid in finding the problem solution.

The following activities are designed to help students develop skills in translating word problems.

SAMPLE ACTIVITY: List the symbol and the word for each operation.

	Symbol for Operation	Word for Operation
(1) a number less seven		subtraction
(2) the difference		
(3) a number increased by eight		
(4) 7 percent of 60		
(5) twice the amount		
(6) sum of their ages		
(7) decreased by		
(8) 80'' board cut in half		
(9) seven hours more than Fred worked		
(10) how many dollars per day?		

SAMPLE ACTIVITY: Have students take some verbal sentences and translate them into mathematical sentences. This activity shows how the translation from narrative language into the language of mathematics involves ordering.

Words	1st Translation	Mathematics
(1) John is 2 years older than Mary.	John's age is Mary's age plus 2.	J = M + 2
(2) For 2 hours, he went many miles at a particular speed.	Number of miles is found by multiplying his speed by 2.	M = 2 x S

Identifying Sufficient, Insufficient, and Extraneous Information

In a mathematics problem, one sometimes finds that there is not enough information provided to solve the problem. Contrastingly, there is sometimes more information provided than is needed to solve the problem. Students need to be able to determine if sufficient information is given and if certain information is extraneous. The following two activities are designed to increase student awareness of insufficient and extraneous information.

SAMPLE ACTIVITY: Give some word problems; ask students whether there is enough information to answer the questions.

SAMPLE ACTIVITY: Have students indicate, by crossing out, underlining, or circling, the extraneous information in word problems.

Using General Problem-Solving Strategies

In order to enhance a student's problem-solving skills, teachers may need to help students develop some systematic approach for attacking word problems. The activities that follow help students develop their own approaches toward problem solving.

SAMPLE ACTIVITY: Using a word problem, guide students through the following strategy:

(1) Scan the material for the purpose of locating specific information.

(2) Reread for important facts.

(3) Interpret clue words.

(4) Decide on an order, if more than one operation is used.

(5) Check to see if there is enough information.

(6) Get a trial answer.

(7) Ask if the solution is reasonable.

(8) Check the answer. If it is wrong, start the process over.

SAMPLE ACTIVITY: Give students various types of word problems to solve, and have them explain their own steps to getting an answer.

STUDY TECHNIQUES

It is important for students to develop good study techniques, which will help them to locate and to understand specific information in factual materials. These techniques require the gathering, organizing, and understanding of materials to be read. When students consistently use good study techniques, they become independent learners. Techniques that are often helpful in this area are:

- adjust reading rate to the material
- read with paper and pencil
- utilize book organization
- use a mathematics dictionary
- use a study formula

ADJUSTING READING RATE TO THE MATERIAL

Students must learn to adjust their reading rate according to the type of material read. Some material can be skimmed; however, generally mathematics material should be read in a careful and precise manner. Symbols hold a quantity of information, and students should understand that it takes longer to read and understand them than it takes to read and understand nonsymbolic terms. The activities that follow can emphasize to students the importance of adjusting their reading rates.

SAMPLE ACTIVITY: When the students have read a passage, involve them in a contest to find all the mathematics words they can in a certain time limit. Then give them a time limit to find additional information. Finally, have them close their books and see how much they remember. As the students recall information, write this information on the board. Now, have the student go back to their books and check to see how much information was not remembered from the skimming activity.

SAMPLE ACTIVITY: Assign a page in a mathematics book to be skimmed by the students. After they skim it, have them summarize what they have learned. Next, have them read the same material carefully and summarize it. When this is finished, have them compare what they comprehended when they skimmed to what they comprehended when they read carefully.

SAMPLE ACTIVITY: Assign a page of narrative material and also a page of mathematical material. Have students compare reading time and content extracted.

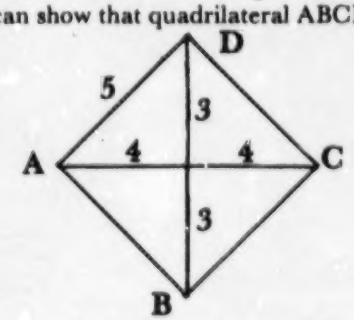
READING WITH PAPER AND PENCIL

Students should develop the habit of reading mathematics material with paper and pencil. Mathematics textbooks generally include concepts, examples, and procedures that must be understood thoroughly before students can proceed with problem solving. Often examples need to be diagrammed or worked through on paper to ensure understanding of the concept. Reading with paper and pencil can aid students in the internalization, comprehension, and recall of information. It allows the readers to complete number sentences, to perform indicated computations, to write questions and answers, and to draw diagrams that will aid their thinking process while they are reading.

The following activities illustrate ways teachers can help students become aware of the importance of reading with paper and pencil.

SAMPLE ACTIVITY: Use passages such as the following to alert students to the importance of using paper and pencil when reading:

> The converse of the Pythagorean Theorem states that if the sum of the squares of the lengths of two sides of a triangle is equal to the square of the length of the third side, then the triangle is a right triangle. For example, a triangle with sides of lengths 3, 4, and 5 is a right triangle, while a triangle with sides of lengths, 2, 2, and 4 is not a right triangle. Using this theorem, one can show that quadrilateral ABCD is a rhombus.



SAMPLE ACTIVITY: Have students read with paper and pencil the section of the textbook they are currently studying. Ask some of the students to discuss how they used their paper and pencil and how it helped their studying. Discuss with the class how you would use paper and pencil with that section.

UTILIZING BOOK ORGANIZATION

Students need to learn what information is available to them in their mathematics textbook and where it can be located. To get the most benefit from their mathematics textbooks, students should become familiar with the format of the pages and chapters. Sometimes different emphasis is given different material by the way it is presented on a page. For example, information can be emphasized by boldface type, italics, boxes, and color codes or pointed out by using a star, an arrow, or an asterisk. The chapter format is the organization of a chapter, such as titles, subtitles, references, and diagrams. Familiarity with the chapter format can aid students in locating information. The following activities provide ways in which teachers can guide their students to an awareness and understanding of their textbook parts and format.

SAMPLE ACTIVITY: Make a list of locational type questions and conduct a class session where students look for information. The class can be divided into two teams. One question is asked at a time, and the students look for the answer. When an answer is found by a team, it is shown to the entire class and that team receives a point.

Example questions:

- On what pages did you find information about absolute value?
- What is the title of Chapter 12?
- How many subtitles are in Chapter 1?
- Who is the author of the book?
- What is the copyright date of your book?
- Where is the glossary located?
- What information can you find about the cover?
- On what page is the chapter summary for Chapter 3?
- Give the name of the publisher.
- Where is the preface located?
- Locate the tables in your book and give the uses of each.

- Are answers to all problems given on the answer pages?
- Which problems are answered (odd, even, etc.) and where are the answers located?

SAMPLE ACTIVITY: If students find a mathematical word they do not know, have them check the page to see if the definition is given. If it is not, then have them check the preceding pages and the glossary. Have the students complete the following table using their textbooks:

Word	Page on Which Definition was Found	Method Used for Presentation of Definition

SAMPLE ACTIVITY: Before beginning a chapter, guide the students through an activity where questions such as the following are asked:

- What is the title of this chapter?
- What are some of the subtitles?
- Read some of the information that is highlighted in some way.
- What are the different ways in which the information is highlighted?
- How long is the chapter?
- Are there references made to other materials?

- What equipment, such as rulers, compasses, strings, protractors, and straight edges, is needed?
- On what pages did you find information about using the above equipment?
- Are there pictures, diagrams, and charts in this chapter?
- What kinds of information do pictures, charts, and diagrams provide?
- Based on your survey of this chapter, what do you think we are going to learn?

USING A MATHEMATICS DICTIONARY

Students should become familiar with a mathematics dictionary by learning how it is organized and what type of information is available in it. The activity that follows provides a way of helping students become aware of the contents and organization of such a dictionary.

SAMPLE ACTIVITY: Duplicate, on acetate, pages from a mathematics dictionary and project them on a screen for the purpose of showing the students how to use the dictionary. The following guidelines can be used:

(1) Show that the listings in a mathematics dictionary, like any other dictionary, are in alphabetical order.

(2) Show one page that contains illustrations.

(3) Show a few of the terms listed.

(4) Show, by underlining, the key words and phrases in some of the definitions.

(5) Show the kind of information found in the appendix:

- Table of Common Logarithms
- Table of Trigonometric Functions
- Table of Compound Interest

- American Experience Table of Mortality
- Table of Squares, Cubes, Square Roots, and Cube Roots of Integers
- Denominate Numbers
- Differentiation Formulas
- Greek Alphabet
- Trigonometric and Hyperbolic Functions

USING A FORMULA FOR STUDY

Sometimes formulas for study can be useful to help students organize their thoughts. One systematic method of studying textbook material is called SQ3R*. This study method consists of the following steps:

- *Survey:* Skim through the material to get an idea of its content and organization.
- *Question:* Form questions about the material.
- *Read:* Read the material; keep in mind the answers to the questions as they are found.
- *Recite (Rewrite):* Recite (rewrite) the important points of the material and the answers to the questions.
- *Review:* Review the answers to the questions and other information learned from the material.

Individual students can use a study formula such as the SQ3R to improve their learning habits and to make the best use of their study time. The following activity illustrates how the SQ3R study method could be used by students.

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*Arnold Burron, and Amos Claybaugh, *Using Reading to Teach Subject Matter*. (Columbus, Ohio: Charles E. Merrill Books, Inc., 1975.)

SAMPLE ACTIVITY:

(1) *Survey:* Locate the assigned reading; locate the exercises to be worked for the assigned material; look at the pictures, maps, charts, and graphs; and find the main topics of the material. Scan for important vocabulary, and look at all highlighted and boxed material.

(2) *Question:* Turn the main topics and headings into questions; find those questions that the material asks.

(3) *Read:* Make an active search to answer these questions.

(4) *Recite (Rewrite):* Put the main ideas of the answers you found in your own words.

(5) *Review:* Try to recall main points and subpoints.

VOLUNTARY MATHEMATICAL READING

One way to encourage better reading of mathematics material is to stimulate student interest in the voluntary reading of mathematics-related material. This reading can vary from historical material to more technical mathematics-related books. Almanacs contain mathematics applications and provide interesting reading. Many trade books are available which involve mathematics applications. These books can be used for information and motivational purposes, and teachers should be aware of their varying difficulty levels. One way teachers can stimulate interest in voluntary reading is to establish a book table where students can go when they complete their other work.

The activities that follow indicate some of the ways interest might be stimulated in the reading of mathematics-related books.

SAMPLE ACTIVITY: Give the students names of mathematicians and a list of materials for locating information about the mathematicians. Allow time for research. Encourage each student to dress up in a costume and imitate the person he or she studied. A "banquet" can be held during one or two class periods. At the banquet, the "mathematicians" will introduce themselves and each will talk briefly about one of his or her accomplishments.

SAMPLE ACTIVITY: As a follow-up activity to a mathematics unit in an elementary class, have the students read mathematics-related books. For titles of some of these books, see Suggested Books for Voluntary Reading, page 35.

SUGGESTED BOOKS FOR VOLUNTARY READING

Adler, Irving and Adler, Ruth. *Directions & Angles.* Scranton: John Day, 1969, (Grades 3-6)

Bedini, Silvio. *Benjamin Banneker.* New York: Scribner, 1972 (Grades 9-12)

Bell, Eric T. *Men of Mathematics.* New York: Simon & Schuster, 1937. (Grades 8 up)

Feltser (Ed by Children's TV Workshop). *The Sesame Street Book of Numbers.* Preschool Press, 1971. (Grades K-6)

Forsee, Aylesa. *Albert Einstein.* New York: Macmillan, 1967. (Grades 7 up)

Hammontree, Marie. *Albert Einstein: Young Thinker.* Indianapolis: Merrill, 1960. (Grades 3-7)

Holden, Alan. *Shapes, Space, & Symmetry.* New York: Columbia University Press, 1971. (Grades 5-7)

Hunt, Leslie L. *Twenty-Five Kites that Fly.* New York: Dover Publications, Inc., 1971 pap.

Juster, Norton. *The Dot and the Line: A Romance in Lower Mathematics.* New York: Random House, 1977 pap.

Knight, David. *Issac Newton, Mastermind of Science.* New York: Watts, 1961. (Grades 7-12)

Lathan, J.L. *Carry on Mr. Bowditch.* Boston: Houghton-Mifflin, 1973. (Grades 6 up)

Luce, Marnie. *Zero is Something.* Minneapolis: Lerner Publications Co., 1969. (Grades 3-6)

Myller, Rolf. *How Big is a Foot.* New York: Antheneum, 1972 pap. (Grades 1-5)

Patterson, Lillie G. *Benjamin Banneker: Genius of Early America.* Nashville: Abingdon, 1978. (Grades 3-7)

Russell, Solveig. *Lines & Shapes.* New York: Walch, Henry Z., 1965. (Grades K-3)

Schaaf, William L. *Carl Friedrich Gauss: Prince of Mathematicians.* New York: Watts, 1964. (Grades 7 up)

Schultz, Pearle and Schultz, Harry. *Issac Newton: Scientific Genius.* New Canaan: Garrad, 1972. (Grades 4-6)

Stonaker, Frances B. *Famous Mathematicians.* Philadelphia: Lippincott, 1966. (Grades 4-9)

Westcott, Alvin M. and Schluep, John. *Fun with Timothy Triangle.* Mankato: Oddo Publishing Inc., 1970. (Grades 4 up)

RESOURCE MATERIALS FOR TEACHERS

Bendrick, Jeanne and Levin, Marcia. *Math Illustrated Dictionary.* McGraw-Hill Book Co., 1965.

Byrne, Mary Ann; Hater, Mary Ann; and Kane, Robert B. *Helping Children Read Mathematics.* New York: American Book Co., 1974.

Byrne, Mary Ann; Hater, Mary Ann; and Kane, Robert B. *Learning to Read Mathematics: Olive Target, Green Target, Aqua Target.* New York: American Book Company, 1973.

Earle, Richard A. *Teaching Reading and Mathematics.* Newark, Delaware: International Reading Association, 1976 (NCTM).

Fay, Leo, and Jared, Lee Ann. *Reading in the Content Fields: An Annotated Bibliography.* Newark, Delaware: International Reading Association, 1975.

Karvsh, William. *The Crescent Dictionary of Mathematics.* New York: The Macmillan Co., 1962.

Olson, Arthur V. and Ames, Wilbur. "Comprehension Skills in Reading," *Teaching Reading Skills in Secondary Schools.* New York: Harper-Row, 1972.

Phillips, E.R., and Uprichard, A.E. "The Correlation of Selected Mathematical Measure with Problem Solving Ability." *Florida Journal of Education Research,* 1974, 16, 3-11.

Rayburn, Elaine. *An Exploration of the Cloze Procedure in Arithmetic Reading.* (Paper presented at the annual meeting of the Texas State Council of IRA), 1974.

Reading Mathematics. Atlanta, Georgia: Georgia State Department of Education, 1974.

Robinson, Alan. *Teaching Reading and Study Strategies: The Content Areas.* Boston: Allyn and Bacon, 1978.

Robinson, Alan and Thomas, Ellen (Editors). *Fusing Reading Skills and Content.* Newark, Delaware: International Reading Association, 1969.

Smith, Nila B. *Be A Better Reader.* Englewood Cliffs, New Jersey: Prentice-Hall, Inc., 1968.

Thomas, Ellen and Robinson, Alan. *Improving Reading in Every Class*. Boston: Allyn and Bacon, 1973.

West, Gail B. *Teaching Reading Skills in Content Areas: A Practical Guide to the Construction of Student Exercises*. Orlando, Florida: Sandpiper Press, Inc., 1974.

COMPLIANCE STATEMENT

TITLE VI, CIVIL RIGHTS ACT OF 1964; THE MODIFIED COURT ORDER, CIVIL ACTION 5281, FEDERAL DISTRICT COURT, EASTERN DISTRICT OF TEXAS, TYLER DIVISION

Reviews of local education agencies pertaining to compliance with Title VI Civil Rights Act of 1964 and with specific requirements of the Modified Court Order, Civil Action No. 5281, Federal District Court, Eastern District of Texas, Tyler Division are conducted periodically by staff representatives of the Texas Education Agency. These reviews cover at least the following policies and practices:

(1) acceptance policies on student transfers from other school districts;
(2) operation of school bus routes or runs on a non-segregated basis;
(3) nondiscrimination in extracurricular activities and the use of school facilities;
(4) nondiscriminatory practices in the hiring, assigning, promoting, paying, demoting, reassigning, or dismissing of faculty and staff members who work with children;
(5) enrollment and assignment of students without discrimination on the basis of race, color, or national origin;
(6) nondiscriminatory practices relating to the use of a student's first language; and
(7) evidence of published procedures for hearing complaints and grievances.

In addition to conducting reviews, the Texas Education Agency staff representatives check complaints of discrimination made by a citizen or citizens residing in a school district where it is alleged discriminatory practices have occurred or are occurring.

Where a violation of Title VI of the Civil Rights Act is found, the findings are reported to the Office for Civil Rights, Department of Health, Education and Welfare.

If there is a direct violation of the Court Order in Civil Action No. 5281 that cannot be cleared through negotiation, the sanctions required by the Court Order are applied.

TITLE VII, CIVIL RIGHTS ACT OF 1964; EXECUTIVE ORDERS 11246 AND 11375; TITLE IX, 1973 EDUCATION AMENDMENTS; REHABILITATION ACT OF 1973 AS AMENDED; 1974 AMENDMENTS TO THE WAGE-HOUR LAW EXPANDING THE AGE DISCRIMINATION IN EMPLOYMENT ACT OF 1967; AND VIETNAM ERA VETERANS READJUSTMENT ASSISTANCE ACT OF 1972 AS AMENDED IN 1974.

It is the policy of the Texas Education Agency to comply fully with the nondiscrimination provisions of all federal and state laws and regulations by assuring that no person shall be excluded from consideration for recruitment, selection, appointment, training, promotion, retention, or any other personnel action, or be denied any benefits or participation in any programs or activities which it operates on the grounds of race, religion, color, national origin, sex, handicap, age, or veteran status (except where age, sex, or handicap constitute a bona fide occupational qualification necessary to proper and efficient administration). The Texas Education Agency makes positive efforts to employ and advance in employment all protected groups.